Lesson 9-2: Sine and Cosine Ratios

Using the hypotenuse

Yesterday we discovered how to use the ratio of the legs of a single right triangle and one of the non-right angles to determine information about the triangle. This ratio is called the tangent ratio. Today we're going to learn about two other ratios involving one leg and the hypotenuse.

Sine and Cosine Ratios

Given one of the non-right angles of a right triangle, either leg relates to the hypotenuse in very useful ratios:

Sine ratio: $\sin A = \frac{opposite}{hypotenuse}$

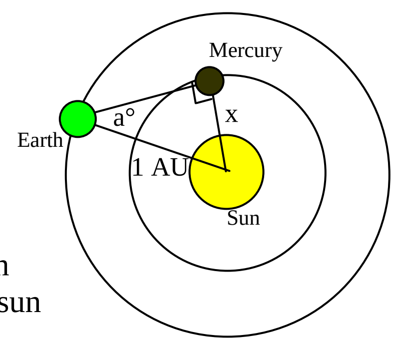
Cosine ratio: $\cos A = \frac{adjacent}{hypotenuse}$

SOH-CAH-TOA

We now have three trigonometric ratios to keep track of: sine, cosine and tangent. There is a silly saying that is often used to help keep these straight. It goes SOH-CAH-TOA and stands for:

SOH: **S**ine **O**pposite over **H**ypotenuse
CAH: **C**osine **A**djacent over **H**ypotenuse
TOA: **T**angent **O**pposite over **A**djacent

How far is Mercury from the Sun?



The book gives a great example of how this stuff has been used. Back around 1500 a man named Copernicus used the sine ratio to figure out how far Mercury (and any planet closer to the sun than the earth) is from the sun. He waited until Mercury was furthest from the sun (from earth's perspective) and used the sine ratio to determine its distance from the sun. Check out this diagram for an example. One *AU* is defined as the distance from the earth to the sun which averages about 93 million miles.

Inverse sine and cosine

Just like the tangent ratio, sine and cosine each have an inverse:

$\sin^{-1}(\frac{opp}{hyp})$: The angle whose sine is $\frac{opp}{hyp}$.

$\cos^{-1}(\frac{adj}{hyp})$: The angle whose cosine is $\frac{adj}{hyp}$.

These are used and determined exactly like the inverse of the tangent ratio.

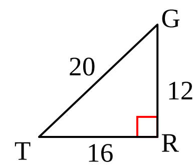
The trig tables again

Quickly turn back to page 731 of your text to the trigonometric tables we looked at yesterday. You will find (in addition to the tangent column) a column each for sine and cosine values. Just for giggles, compare the sine value for 30° to the cosine value for

60°. What do you notice? Do the same for several other complementary angles. You have discovered one of the first "trigonometric identities:" $\sin x = \cos(90 - x)$. You will work with the trig identities much more next year.

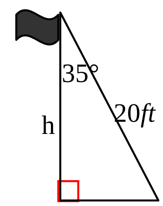
Examples

1. Use the triangle to find sin *T*, cos *T*, sin *G*, and cos *G*.



$$\sin T = \frac{opp}{hyp} = \frac{12}{20}; \cos T = \frac{adj}{hyp} = \frac{16}{20}; \sin G = \frac{opp}{hyp} = \frac{16}{20}; \cos G = \frac{adj}{hyp} = \frac{12}{20}$$

2. A 20 *ft* wire supporting a flagpole forms a 35° angle with the flagpole. To the nearest foot, how high is the flagpole?



   We are looking for *h* which is adjacent to the angle. Use cosine.

   $$\cos 35 = \frac{h}{20}; h = 20 \cdot \cos 35 = 16.38 \approx 16\ ft$$

3. A right triangle has a leg 1.5 unit long and a hypotenuse 4.0 units long. Find the measures of its acute angles to the nearest degree.

   Pick the angle with the 1.5 unit leg opposite it. That means we'll need to use the inverse of the sine ratio: $m\angle A = \sin^{-1}(\frac{1.5}{4.0}) = 22.02 \approx 22°$

   Now pick the angle with the 1.5 unit long leg adjacent to it. That means we'll use the inverse of the cosine ratio: $m\angle B = \cos^{-1}(\frac{1.5}{4.0}) = 67.97 \approx 68°$

Homework Assignment

p. 479 #1-17, 22-24, 26, 28, 37, 43, 45-47